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The VOSIM Signal Spectrum

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ABSTRACT

The VOSIM sound synthesis system is based on a signal consisting of a series of $\sin^2$ pulses with a staircase-shaped envelope. In this article the spectrum of such a signal is derived.

INTRODUCTION

Werner Kaegi's first publications (Kaegi, 1973, 1974) on a sound synthesis system which has since been given the name VOSIM appeared in 1973 and 1974. The system has been

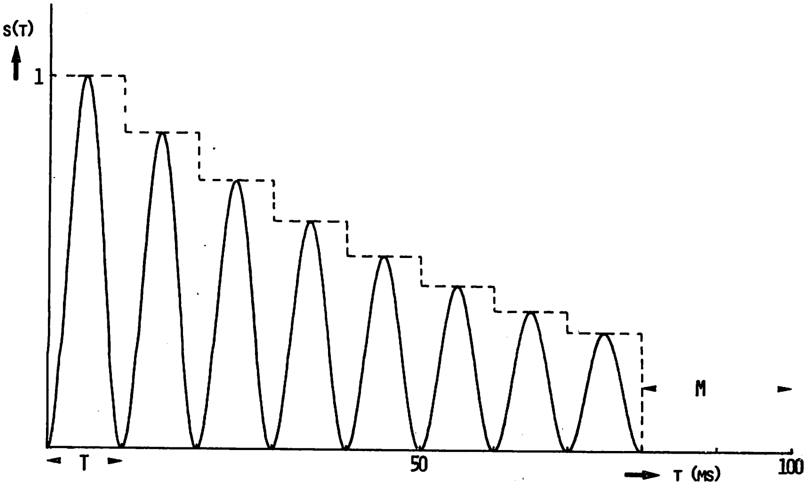

Fig. 1. The VOSIM time function (N = 8, b = 0.85, T = 10 ms).

elaborated upon since, attention also having been paid to its signal-theoretic aspects. These aspects will be examined more closely in this article.

The most general form of the VOSIM signal can be seen in fig. 1.

It consists of a series of N $\sin^2$ pulses with a pulse-width T, followed by a delay M. The signal has a staircase-shaped envelope, i.e. the first pulse has an amplitude A and each succeeding pulse has an amplitude which is a factor b smaller that that of its predecessor. The reasons for using this type of envelope instead of an exponential one were discussed in an earlier publication (Tempelaars et al., 1976).

The model has turned out to be extremely flexible. There are several reasons for this. It is particularly easy to regulate the pitch of a periodically repeated VOSIM signal. This is because the period duration T′ is equal to

$$T' = N \cdot T + M \qquad (1)$$

This means that the repetition frequency f′ and thus the pitch can be varied by altering the duration of the delay M. When the values of N and T remain the same, the connection between f′ and M is:

$$f' = \frac{1}{N \cdot T + M} \quad \text{(see fig. 2)} \qquad (2)$$

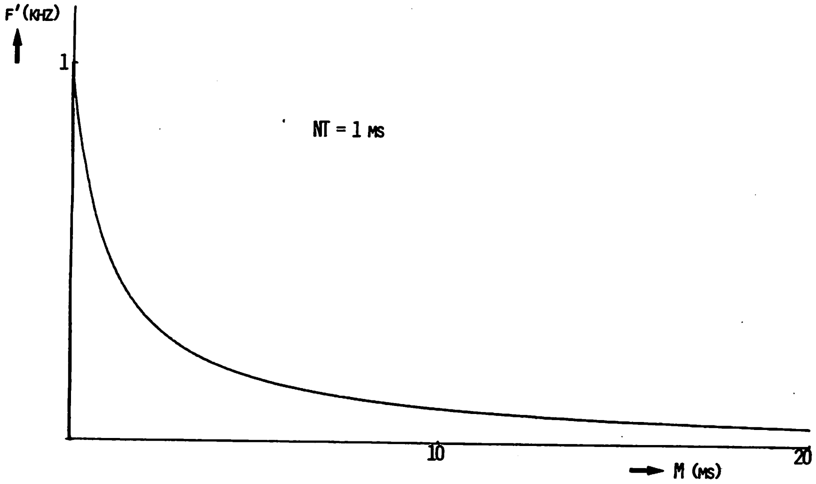

Fig. 2. The connection between f′ and M.

What is more, M can be modulated. In the case of rapid, sinusoidal modulation, sidebands occur; in the case of slow modulation, vibrato occurs; in the case of random modulation, noise is introduced. When the pulse-width T and the amplitude A are varied, the number of possibilities increases.

Using one, and in certain cases two synchronized VOSIM signals, it has proved possible to produce not only speech sounds but also musical sounds (including flute, clarinet, bassoon, horn, trumpet, harpsichord, piano etc.). Three synchronized VOSIM signals are required to produce string sounds.

In this first theoretical analysis we shall restrict ourselves to the signal illustrated in fig. 1, N, T and b being constant. We calculate the spectrum of a single and of a periodically repeated version of this signal. In the latter case the spectrum turns out to have a formant structure which is determined by the pulse-width of the $\sin^2$ pulses; this fact partly accounts for the feasibility of the model for producing the said sounds.

THE TIME FUNCTION

The time function s(t) of a VOSIM signal (see fig. 1) is the product of a sinusoidal signal p(t) and a staircase-shaped envelope f(t):

$$s(t) = f(t) \cdot p(t) \qquad (3)$$

In this:

$$f(t) = u(t) - b^{N-1}\, u(t - NT) + (b - 1) \sum_{n=1}^{N-1} b^{n-1} \cdot u(t - nT) \qquad (4)$$

and

$$p(t) = \tfrac{1}{2}(1 - \cos \Omega t) \quad (= \sin^2 2\Omega t) \qquad (5)$$

with

$$u(t)\text{: the step function, } u(t) = \begin{cases} 1 \text{ if } t \geq 0 \\ 0 \text{ if } t < 0 \end{cases}$$

b : the attenuation coefficient
N : the number of $\sin^2$ pulses
T : the duration of the $\sin^2$ pulses
Ω : their frequency

$$\left.\begin{matrix}T\text{ : the duration of the } \sin^2 \text{ pulses} \\ \Omega\text{ : their frequency}\end{matrix}\right\} T = \frac{2\pi}{\Omega}$$

(For the sake of convenience we assume the amplitude A of the first pulse to be equal to 1.)

THE SPECTRUM

If $F(\omega)$ is the Fourier transform of f(t) and $S(\omega)$ that of s(t), the following applies:

$$S(\omega) = F\{s(t)\} = F\{\tfrac{1}{2}f(t) \cdot (1 - \cos \Omega t)\} = \tfrac{1}{2}\{F(\omega) - \tfrac{1}{2}F(\omega - \Omega) - \tfrac{1}{2}F(\omega + \Omega)\} \qquad (6)$$

For $F(\omega)$ we find:

$$F(\omega) = \frac{1 - e^{-j\omega T}}{j\omega} \cdot \frac{1 - b^{N} e^{-j\omega NT}}{1 - b \cdot e^{-j\omega T}} \tag{7}$$

and from it for S(ω):

$$S(\omega) = \frac{j(1 - e^{-2\pi j\frac{\omega}{\Omega}})}{2\omega(\frac{\omega^2}{\Omega^2} - 1)} \cdot \frac{1 - b^{N} e^{-2\pi jN\frac{\omega}{\Omega}}}{1 - b \cdot e^{-2\pi j\frac{\omega}{\Omega}}} \tag{8}$$

This expression is not defined for $\omega=0$ and $\omega=\Omega$; we determine $S(0)$ and $S(\Omega)$ by using (6):

a. $S(0) = \frac{1}{2}\{F(0) - \frac{1}{2}F(-\Omega) - \frac{1}{2}F(\Omega)\} = \frac{1}{2}F(0)$ since $F(\pm\Omega) = 0$ following from (7).

For F(0) we have to determine:

$$\lim_{\omega \to 0} \frac{1 - e^{-j\omega T}}{j\omega} = \lim_{\omega \to 0} \frac{1 - \cos \omega T + j \cdot \sin \omega T}{j\omega}$$

$$= \lim_{\omega \to 0} \left(\frac{T}{j} \, \frac{1 - \cos \omega T}{a T} + T \, \frac{\sin \omega T}{\omega T} \right) = T$$

We must also take into account the possibility that $b = 1$:

$$\frac{1 - b^{N}}{1 - b} = \sum_{n=0}^{N-1} b^{n}, \text{ for } b = 1 \text{ this is equal to } N.$$

Collating this, we find for F(0):

$$F(0) = \begin{cases} T\dfrac{1 - b^{N}}{1 - b} & \text{if } b < 1 \\ TN & \text{if } b = 1 \end{cases} \tag{9}$$

and for S(0):

$$S(0) = \begin{cases} \frac{1}{2}T\dfrac{1 - b^{N}}{1 - b} & \text{if } b < 1 \\ \frac{1}{2}TN & \text{if } b = 1 \end{cases} \tag{10}$$

b. $S(\Omega) = \frac{1}{2}\{F(\Omega) - \frac{1}{2}F(0) - \frac{1}{2}F(2\Omega)\} = -\frac{1}{4}F(0)$

or

$$S(\Omega) = \begin{cases} -\frac{1}{4}T\dfrac{1 - b^N}{1 - b} & \text{if } b < 1 \\ -\frac{1}{4}TN & \text{if } b = 1 \end{cases} \tag{11}$$

THE MAGNITUDE SPECTRUM AND THE PHASE SPECTRUM

We can split up $S(\omega)$ into the magnitude spectrum $|S(\omega)|$ and the phase spectrum $\Phi(\omega)$:

$$S(\omega) = |S(\omega)| \cdot e^{j\Phi(\omega)} \tag{12}$$

The magnitude spectrum is equal to

$$|S(\omega)| = \left| \frac{\sin \pi \frac{\omega}{\Omega}}{\omega(\frac{\omega^2}{\omega 2} - 1)} \right| \sqrt{\frac{1 - 2b^N \cos 2\pi N\frac{\omega}{\Omega} + b^{2N}}{1 - 2b \cdot \cos 2\pi\frac{\omega}{\Omega} + b^2}} \tag{13}$$

with

$$|S(0)| = \begin{cases} \frac{1}{2}T\dfrac{1 - b^N}{1 - b} & \text{if } b < 1 \\ \frac{1}{2}TN & \text{if } b = 1 \end{cases} \quad \text{and} \quad |S(\Omega)| = \begin{cases} \frac{1}{4}T\dfrac{1 - b^N}{1 - b} & \text{if } b < 1 \\ \frac{1}{4}TN & \text{if } b = 1 \end{cases}$$

The phase spectrum is described by:

$$\Phi(\omega) = tg^{-1} \frac{(1 + b) \sin \pi\frac{\omega}{\Omega} - b^N \sin(2N + 1) \pi\frac{\omega}{\Omega} + b^{N+1} \sin(2N - 1) \pi\frac{\omega}{\Omega}}{(b - 1) \cos \pi\frac{\omega}{\Omega} + b^N \cos(2N + 1) \pi\frac{\omega}{\Omega} - b^{N+1} \cos(2N - 1) \pi\frac{\omega}{\Omega}} \tag{14}$$

Since $S(0)$ and $S(\Omega)$ are real, $\Phi(0) = \Phi(\Omega) = \pm k\pi$ $(k = 0, 1, 2, \ldots)$

A FEW SPECIAL CASES

a. $N = 1$: $$S(\omega) = \frac{j(1 - e^{-j2\pi\frac{\omega}{\Omega}})}{2\omega(\frac{\omega^2}{\Omega^2} - 1)} \tag{15}$$

$$\left|S(\omega)\right| = \left|\frac{\sin \pi \frac{\omega}{\Omega}}{\omega(\frac{\omega^2}{\Omega^2} - 1)}\right| \tag{16}$$

$$\Phi(\omega) = -\pi \frac{\omega}{\Omega} \tag{17}$$

b. *N = 2, b < 1:*

$$S(\omega) = \frac{j(1 - e^{-j2\pi\frac{\omega}{\Omega}})}{2\omega(\frac{\omega^2}{\omega^2} - 1)} (1 + b \cdot e^{-j2\pi\frac{\omega}{\Omega}}) \tag{18}$$

$$\left|S(\omega)\right| = \left|\frac{\sin \pi \frac{\omega}{\Omega}}{\omega(\frac{\omega^2}{\Omega^2} - 1)}\right| \sqrt{1 + 2b \cdot \cos 2\pi \frac{\omega}{\Omega} + b^2} \tag{19}$$

$$\Phi(\omega) = \mathrm{tg}^{-1} \frac{(1 + b) . \sin \pi \frac{\omega}{\Omega} + b^3 \sin 3\pi \frac{\omega}{\Omega} - b^2 \sin 5\pi \frac{\omega}{\Omega}}{(b - 1) \cos \pi \frac{\omega}{\Omega} - b^3 \cos 3\pi \frac{\omega}{\Omega} + b^2 \cos 5\pi \frac{\omega}{\Omega}} \tag{20}$$

c. *b = 1*

$$S(\omega) = \frac{j(1 - e^{-j2\pi N\frac{\omega}{\Omega}})}{2\omega(\frac{\omega^2}{\Omega^2} - 1)} \tag{21}$$

$$\left|S(\omega)\right| = \left|\frac{\sin \pi N \frac{\omega}{\Omega}}{\omega(\frac{\omega^2}{\Omega^2} - 1)}\right| \tag{22}$$

$$\Phi(\omega) = - N\pi \frac{\omega}{\Omega} \tag{23}$$

For purposes of illustration, diagrams of the magnitude spectrum and phase spectrum for the case N = 8, b = 0.85 are included (the time function is shown in fig. 1) in figs. 3 and 4 respectively. There are also diagrams of the time function, the magnitude spectrum and the phase spectrum for the case N = 1 (figs. 5, 6 and 7), for N = 2, b = 0.75 (figs. 8, 9 and 10) and for N = 6, b = 1 (figs. 11, 12 and 13).

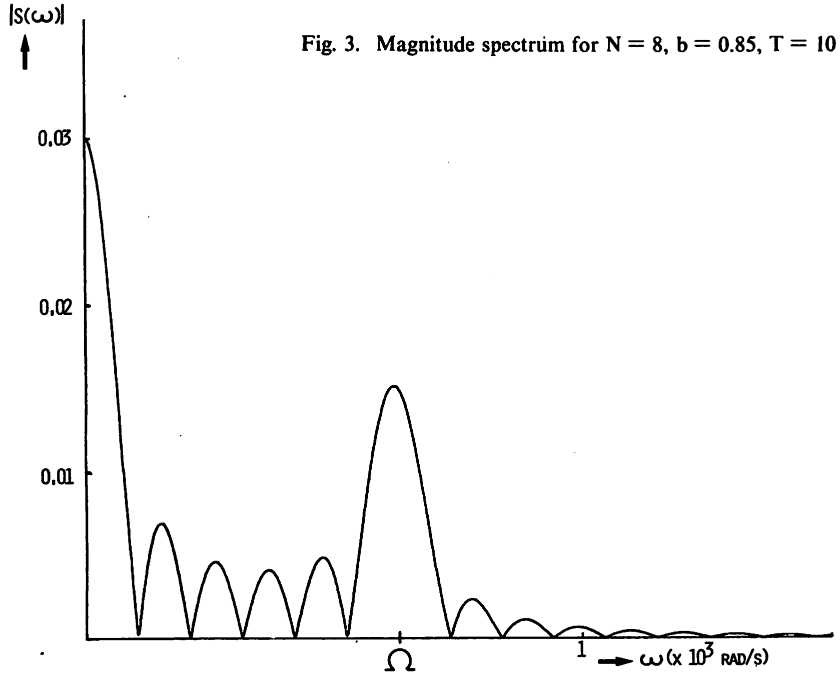

Fig. 3. Magnitude spectrum for N = 8, b = 0.85, T = 10 ms.

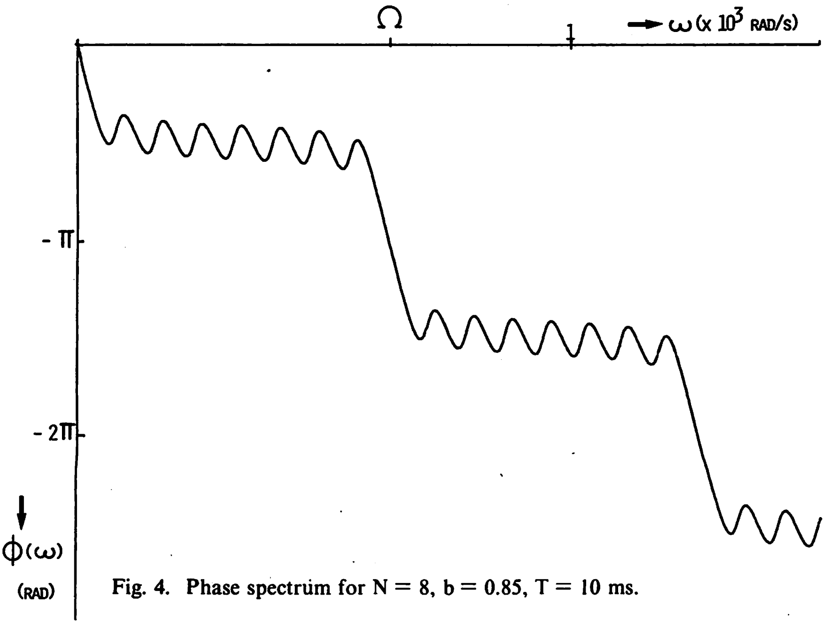

Fig. 4. Phase spectrum for N = 8, b = 0.85, T = 10 ms.

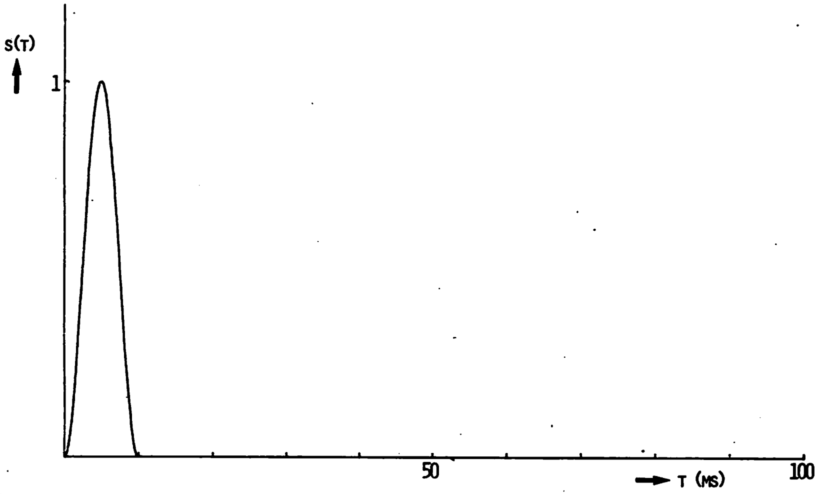

Fig. 5. s(t) for N = 1.

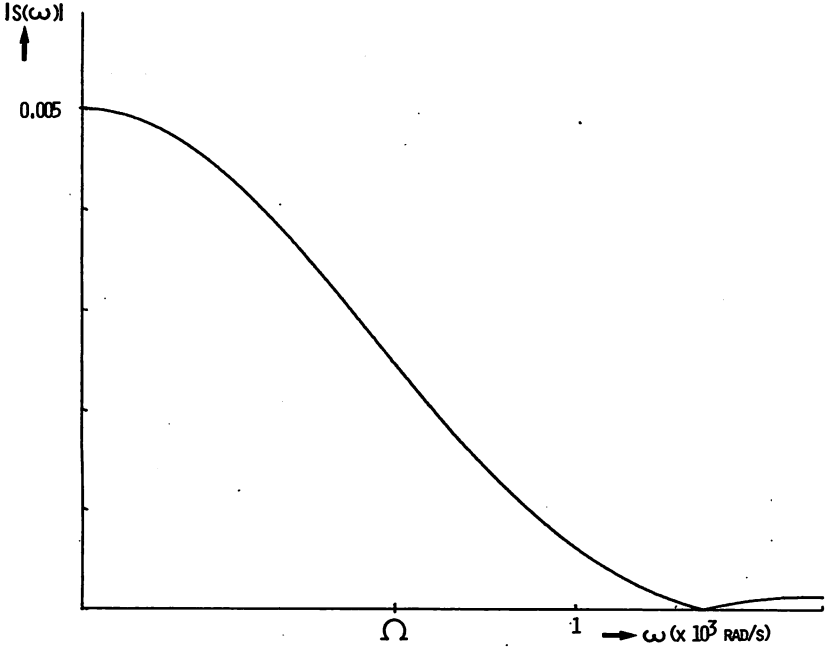

Fig. 6. Magnitude spectrum for N = 1.

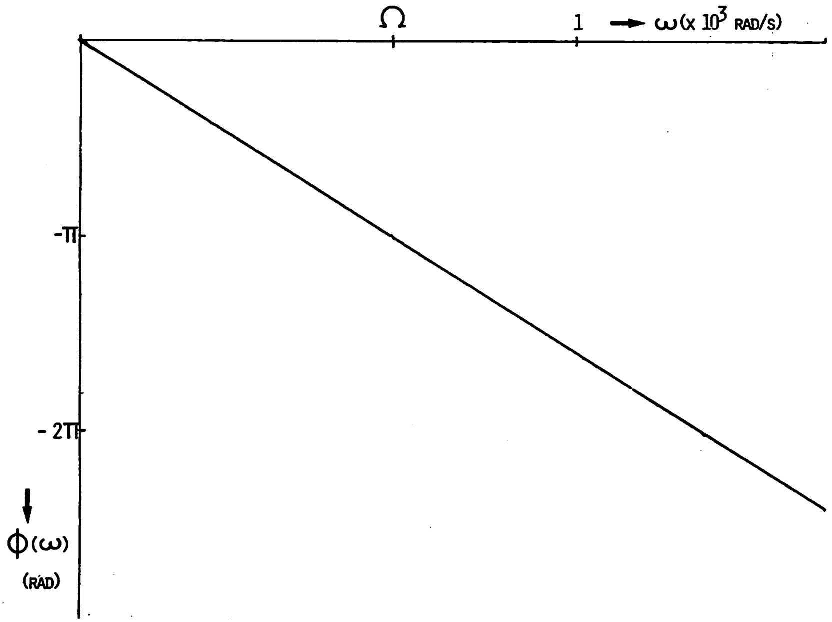

Fig. 7. Phase spectrum for N = 1.

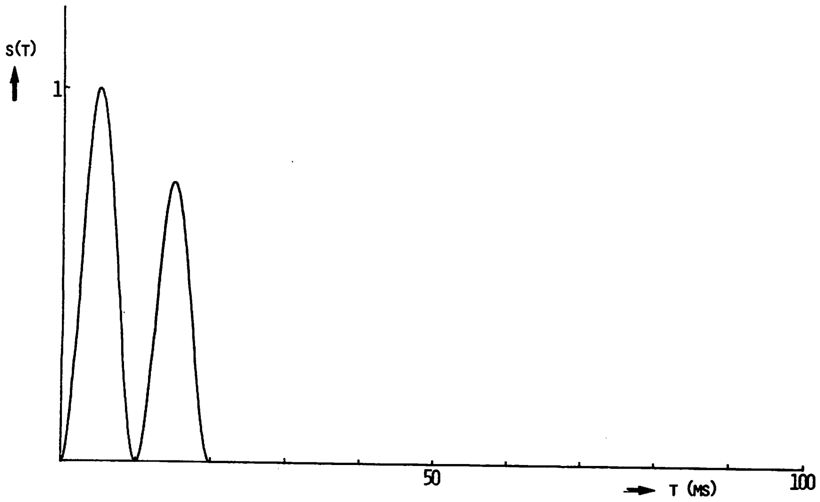

Fig. 8. s(t) for N = 2, b = 0.75.

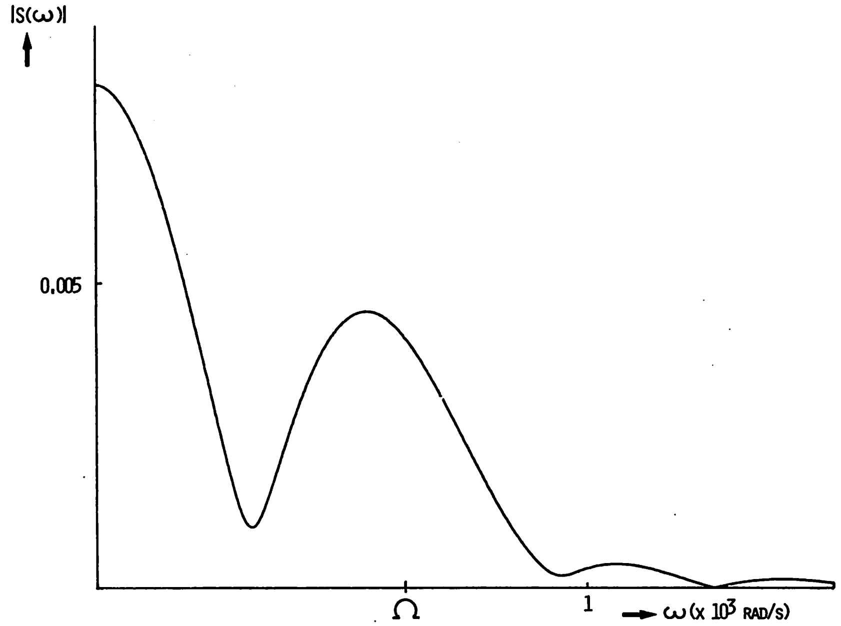

Fig. 9. Magnitude spectrum for N = 2, b = 0.75.

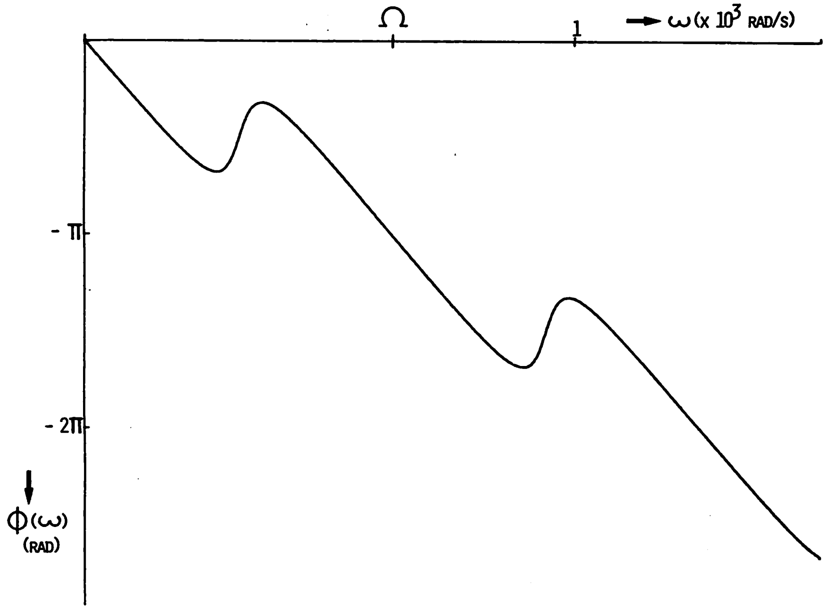

Fig. 10. Phase spectrum for N = 2, b = 0.75.

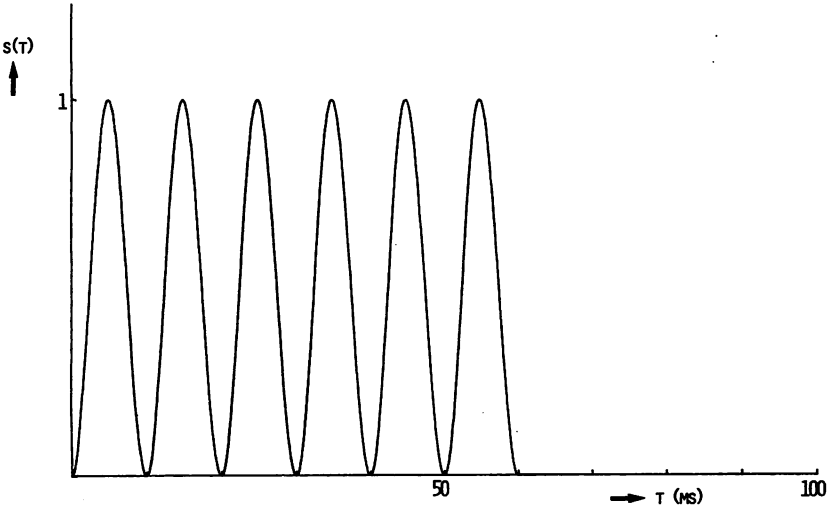

Fig. 11. s(t) for N = 6, b = 1.

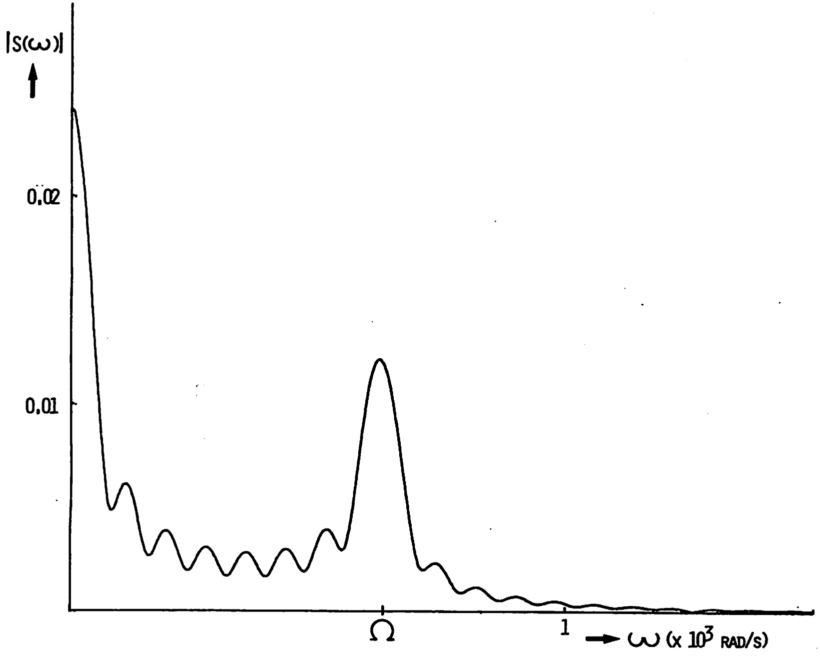

Fig. 12. Magnitude spectrum for N = 6, b = 1.

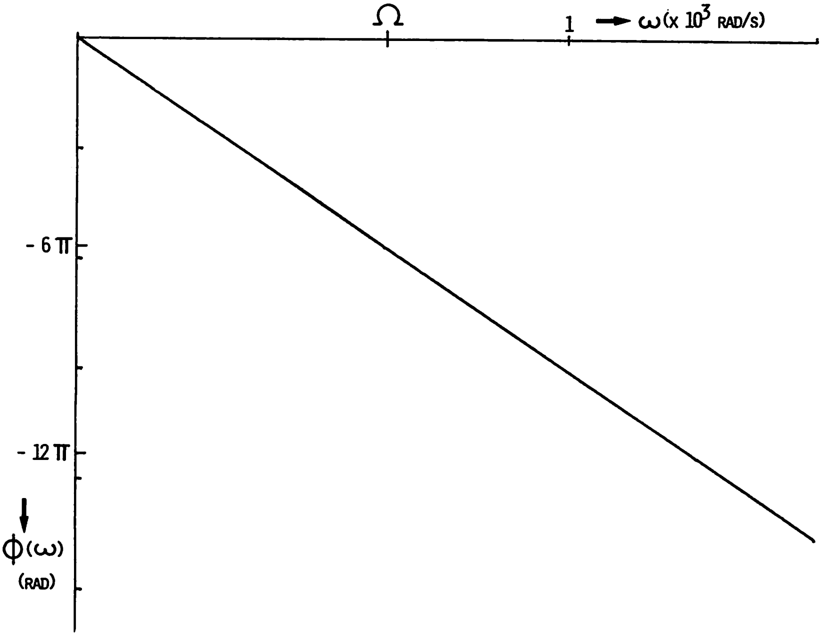

Fig. 13. Phase spectrum for N = 6, b = 1.

THE SPECTRUM OF A PERIODIC VOSIM-SIGNAL

When a signal like the one shown in fig. 1 is repeated periodically, the spectrum changes into a discrete spectrum. The complex coefficients c_n can be derived from the continuous spectrum $S(\omega)$: when the period is T' ($T' > NT$) and the appropriate frequency $\omega'(= 2\pi/T')$, the following applies:

$$c_n = \frac{1}{T'} S(n\omega') \tag{24}$$

The amplitude coefficient C and the phase coefficient can be calculated from this:

$$C_n = 2|c_n| \quad \text{(s(t) is a real function)} \tag{25}$$

$$\Phi_n = \mathrm{tg}^{1} \frac{\mathrm{Im}\,(c_n)}{\mathrm{Re}\,(c_n)} \tag{26}$$

If we apply these rules (abbreviating the quotient $\frac{\omega^1}{\Omega}$ to γ), we arrive at the following results, analogous to those for the continuous spectrum:

$$c_n = \frac{j(1 - e^{-j2\pi n\gamma})}{4\pi n(n^2\gamma^2 - 1)} \cdot \frac{1 - b^N e^{j2\pi Nn\gamma}}{1 - b \cdot e^{j2\pi n\gamma}} \tag{27}$$

$$a_0 = 2c_0 = \frac{2}{T'},\ S(0) = \begin{cases} \gamma \dfrac{1 - b^N}{1 - b} & \text{if } b < 1 \\ \gamma N & \text{if } b = 1 \end{cases} \tag{28}$$

and for the frequency component with number n′, with $n'\omega' = \Omega$ $(n'\gamma = 1)$

$$c_{n'} = \begin{cases} -\tfrac{1}{4}\gamma \dfrac{1 - b^N}{1 - b} & \text{if } b < 1 \\ -\tfrac{1}{4}\gamma N & \text{if } b = 1. \end{cases} \tag{29}$$

$$C_n = \left| \frac{\sin \pi n\gamma}{\pi n(n^2\gamma^2 - 1)} \right| \sqrt{\frac{1 - 2b^N \cos 2\pi N n\gamma + b^{2N}}{1 - 2b \cdot \cos 2\pi n\gamma + b^2}} \tag{30}$$

$$C_{n'} = \begin{cases} \tfrac{1}{2}\gamma \dfrac{1 - b^N}{1 - b} & \text{if } b < 1 \\ \tfrac{1}{2}\gamma N & \text{if } b = 1\ , \end{cases} \tag{31}$$

$$\Phi_n = \mathrm{tg}^{-1} \frac{(1 + b) \sin \pi n\gamma - b^N \sin (2N + 1)\pi n\gamma + b^{N+1} \sin (2N - 1)\pi n\gamma}{(b - 1) \cos \pi n\gamma + b^N \cos (2N + 1)\pi n\gamma - b^{N+1} \cos (2N - 1)\pi n\gamma} \tag{32}$$

In the same way we find for the 'special' cases:

a. $N = 1$

$$c_n = \frac{j\,(1 - e^{-j2\pi n\gamma})}{4\pi n\,(n^2\gamma^2 - 1)} \tag{33}$$

$$C_n = \left| \frac{\sin \pi n\gamma}{\pi n\,(n^2\gamma^2 - 1)} \right| \tag{34}$$

$$\Phi_n = -\pi n\gamma$$

b. $N = 2$, $b < 1$

$$c_n = \frac{j\,(1 - e^{-j2\pi N n\gamma})}{4\pi n\,(n^2\gamma^2 - 1)} \tag{36}$$

$$C_n = \left| \frac{\sin \pi n \gamma}{\pi n (n^2\gamma^2 - 1)} \right| \sqrt{1 + 2b \cdot \cos 2\pi n \gamma + b^2} \tag{37}$$

$$\Phi_n = \mathrm{tg}^{-1} \frac{(1 + b) \sin \pi n \gamma + b^3 \sin 3\pi n \gamma - b^2 \sin 5\pi n \gamma}{(b - 1) \cos \pi n \gamma - b^3 \sin 3\pi n \gamma + b^2 \cos 5\pi n \gamma} \tag{38}$$

c. $b = 1$

$$c_n = \frac{j(1 - e^{-j2\pi N n \gamma})}{4\pi n (n^2\gamma^2 - 1)} \tag{39}$$

$$C_n = \left| \frac{\sin \pi N n \gamma}{\pi n (n^2\gamma^2 - 1)} \right| \tag{40}$$

$$\Phi_n = -N\pi n \gamma \tag{41}$$

The reader is referred to the previous figures for a graphic representation, the continuous lines being regarded as envelopes of the line-spectra. The graduated scale along the vertical axis changes as a consequence of the factor 2/T, (see (24) and (25)).

MISSING HARMONICS

When $b = 1$, the spectrum exhibits a large number of zero points. The harmonics corresponding with these zero points are therefore missing from the spectrum. It follows from (40) that this is the case when

$$\sin \pi N n \gamma = 0$$

or, when

$$N n \gamma = k \ (k = 1, 2, 3, \ldots)$$

This means that the harmonics with numbers n satisfying

$$n = \frac{k}{N\gamma} \tag{42}$$

are missing. Condition (42) can be elaborated as follows:

$$N\gamma = \frac{NT}{T'} = \frac{NT}{NT + M}$$

$$n = \frac{k}{N\gamma} = k\,(1 + \frac{M}{NT})$$

M and NT are durations which can always be written as rational numbers within the given degree of accuracy. The quotient M/NT is rational too, and when divided by the greatest common divisor of M and NT, it can be reduced to the quotient p/q, which cannot be simplified further and in which p and q are whole numbers. In this way we find

$$n = k\,(1 + \frac{p}{q}) = k\,\frac{p + q}{q} \tag{43}$$

Since a 'k' can always be found which is a multiple of the whole number q, it follows that the numbers of the missing harmonics are multiples of p + q. The only exception is the case n = 1. This leads to

$$n = \frac{1}{\gamma} = \frac{T'}{T} = N \cdot \frac{NT + M}{NT} = N \cdot \frac{p + q}{q}$$

Hence, if N/q is a whole number, that particular multiple of p + q will not be missing (after all, this is in fact the 'formant frequency'). In that case the amplitude is, according to (31), equal to

$$C_n = \tfrac{1}{2}\gamma N = \frac{q}{2\,(p + q)}$$

Example:
Assume M/NT = 3/2 (= p/q); the multiples of p + q will then be missing, in other words the harmonics whose numbers are multiples of 5. If in this case N = 2 too, the fifth harmonic itself will not be missing.

CONCLUSION

The above derived expressions for the VOSIM signal spectrum and the relevant diagrams enable anyone interested in experimenting with this sound model to pre-determine the values of T, N and b for a particular desired spectrum. A programme for the programmable calculator HP-67 for calculating magnitude and phase values is available on request.

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